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Simulating Open Quantum Systems with Trapped Ions*

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Abstract. This paper focuses on the possibility of simulating the open system dynamics of a paradigmatic model, namely the damped harmonic oscillator, with single trapped ions. The key idea consists in using a controllable physical system, i.e. a single trapped ion interacting with an engineered reservoir, to simulate the dynamics of other open systems usually difficult to study. The exact dynamics of the damped harmonic oscillator under very general conditions is firstly derived. Some peculiar characteristic of the system's dynamics are then presented. Finally a way to implement with trapped ion the specific quantum simulator of interest is discussed.

Keywords: open quantum systems, quantum computation, trapped ions, non-Markovian dynamics
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1. Introduction

The dynamics of closed systems may be calculated exactly by solving directly the Schrödinger equation. In realistic physical conditions, however, quantum mechanical systems need to be regarded as open systems due to the fact that, as in classical physics, any realistic system is coupled to an uncontrollable environment which influences it in a non-negligible way [1].

Nowadays the interest in the broad field of open quantum systems has notably increased mainly for two reasons. On the one hand, experimental advances in the coherent control of single or few atoms and ions have paved the way to the realization of the first basic elements of quantum computers, c-not [2] and phase

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quantum gates [3]. Moreover, the first quantum cryptographic [4] and quantum teleportation [5] schemes have been experimentally implemented. These technological applications rely on the persistence of quantum coherence. Thus, understanding decoherence and dissipation arising from the unavoidable interaction between the system and its surrounding is necessary in order to implement these new quantum technologies.

On the other hand, one of the most debated aspects of quantum theory, namely the quantum measurement problem, has been recently interpreted in terms of environment induced decoherence [6]. According to this interpretation the emergence of the classical world from the quantum world can be seen as a decoherence process due to the interaction between the system and the environment. For this reason the study of some paradigmatic models of open systems allows to gain new insight in the theory of quantum measurement and in the related fundamental issues of quantum theory.

This paper focus on the possibility of studying experimentally a paradigmatic model of the theory of open quantum system, namely the damped harmonic oscillator or quantum Brownian motion (QBM) model [7–9]. We propose to simulate the system dynamics with single trapped ions coupled to artificial reservoirs. The idea of simulating the dynamics of a given (closed) quantum systems by using other more easily controllable systems was introduced by Feynman in [10]. Following this idea, some experimental schemes for realizing quantum simulators have been proposed in the trapped ion context [11, 12]. In this paper we propose to extend the concept of quantum simulators from closed to open quantum systems. The possibility of realizing an open system quantum simulator stems from the recent experimental achievements in the realization of artificial reservoirs with trapped ions. In this context, indeed, it is possible not only to engineer experimentally an artificial reservoir but also to synthesize both its spectral density and the coupling with the system oscillator [13, 14]. This makes it possible to think of new types of experiments aimed at testing the predictions of fundamental models as the one of quantum Brownian motion (or its high T limit: the famous Caldeira–Leggett model [8]).

The paper is structured as follows. In Section 2 the exact master equation for QBM is presented and an analytical method to solve it is discussed. In Section 3, the time evolution of the mean energy of the particle is studied. In Section 4 the experimental conditions for simulating QBM and observing the peculiar dynamics of the mean energy of the systems are analyzed. Finally, in Section 5 conclusions are presented.

2. Quantum Brownian Motion

2.1. Master Equation

The dynamics of a harmonic oscillator linearly coupled with a quantized reservoir, modeled as an infinite chain of quantum harmonic oscillators, is described, in the secular approximation, by means of the following generalized Master Equation [15]

$$\begin{aligned}\frac{d\rho_S(t)}{dt} &= \frac{\Delta(t)+\gamma(t)}{2}\left[2a\rho_S(t)a^\dagger - a^\dagger a\rho_S(t) - \rho_S(t)a^\dagger a\right] \\ &\quad + \frac{\Delta(t)-\gamma(t)}{2}\left[2a^\dagger\rho_S(t)a - aa^\dagger\rho_S(t) - \rho_S(t)aa^\dagger\right] . \end{aligned} \tag{1}$$

The time dependent coefficients appearing in the Master Equation can be written, to the second order in the coupling strength, as follows

$$\Delta(t) = \int_0^t \kappa(\tau)\cos(\omega_0\tau)d\tau\,, \qquad \gamma(t) = \int_0^t \mu(\tau)\sin(\omega_0\tau)d\tau\,, \tag{2}$$

where $\kappa(\tau)$ and $\mu(\tau)$ are the noise and dissipation kernels, respectively, and ω_0 is the frequency of the system oscillator [1].

The Master Equation (1) is local in time, even if non-Markovian. This feature is typical of all the generalized Master Equations derived by using the time-convolutionless projection operator technique [1] or equivalent approaches such as the superoperatorial one presented in [9, 16]. It is worth noting that the Master Equation (1) is of Lindblad-type as far as the coefficients $\Delta(t) \pm \gamma(t)$ are positive [17].

In what follows we study the time evolution of the heating function $\langle n(t)\rangle$ with n quantum number operator. This operator belongs to a class of observables not influenced by the secular approximation [9,18]. For this reason, in order to calculate the exact time evolution of the heating function, one can use the solution of the approximated Master Equation (1).

2.2. *Time evolution of the Quantum Characteristic Function*

The Master Equation (1) can be solved exactly by using specific algebraic properties of the superoperators [9]. The solution for the density matrix of the system is derived in terms of the quantum characteristic function (QCF) $\chi_t(\xi)$ at time t, defined through the equation [19]

$$\rho_S(t) = \frac{1}{2\pi}\int \chi_t(\xi)\, e^{\left(\xi a^\dagger - \xi^* a\right)} d^2\xi\,. \tag{3}$$

It is worth noting that one of the advantages of this approach is the easiness in calculating the analytic expression for the mean values of observables of interest by means of the relation

$$\langle a^{\dagger m} a^n\rangle = \left(\frac{d}{d\xi}\right)^m \left(-\frac{d}{d\xi^*}\right)^n e^{|\xi|^2/2}\chi(\xi)\Big|_{\xi=0}\,. \tag{4}$$

The exact analytic expression for the time evolution of the heating function can be obtained from the solution of Eq. (1). In the secular approximation the QCF takes the form [9]

$$\chi_t(\xi) = e^{-\Delta_\Gamma(t)|\xi|^2}\chi_0\left[e^{-\Gamma(t)/2}e^{-i\omega_0 t}\xi\right], \tag{5}$$

with χ_0 QCF of the initial state of the system. The quantities $\Delta_\Gamma(t)$ and $\Gamma(t)$ appearing in Eq. (5) are defined in terms of the diffusion and dissipation coefficients $\Delta(t)$ and $\gamma(t)$, respectively, as follows

$$\Gamma(t) = 2\int_0^t \gamma(t_1)\, dt_1\,, \qquad \Delta_\Gamma(t) = e^{-\Gamma(t)}\int_0^t e^{\Gamma(t_1)}\Delta(t_1)dt_1\,. \tag{6}$$

We now focus on the dynamics of the heating function $\langle n(t)\rangle$. Having in mind Eq. (5) and using Eq. (4), one gets the following expression for the heating function

$$\langle n(t)\rangle = e^{-\Gamma(t)}\langle n(0)\rangle + \frac{1}{2}\left(e^{-\Gamma(t)} - 1\right) + \Delta_\Gamma(t)\,. \tag{7}$$

In the next section we will discuss in detail the dynamics of the heating process and we will show the changes in the short time behavior due to the variations of typical reservoir parameters.

3. Time Evolution of the Mean Energy: Lindblad-Type and Non-Lindblad-Type Dynamics

In a previous paper we have presented a theory of heating for a single trapped ion interacting with a natural reservoir able to describe both its short time non-Markovian behavior and the asymptotic thermalization process [20]. Here we focus instead on the case of interaction with engineered reservoirs. In the trapped ion context, it is possible to engineer artificial reservoirs and couple them to the system in a controlled way. Since the coupling with the natural reservoir is negligible for long time intervals [21], this allows to test fundamental models of open system dynamics as the one for QBM we are interested in. By using the analytic solution, one can look for ranges of the relevant parameters of both the reservoir and the system in correspondence of which deviations from Markovian dissipation become experimentally observable.

In the experiments on artificially engineered amplitude reservoirs [14] the high temperature condition $\hbar\omega_0/KT \ll 1$ is always satisfied. For this reason here we concentrate on this regime of the parameters. We assume an Ohmic reservoir spectral density with Lorentz–Drude cut-off

$$J(\omega) = \frac{2\omega}{\pi}\,\frac{\omega_c^2}{\omega_c^2+\omega^2}, \tag{8}$$

with ω_c cut-off frequency.

For times much smaller then the thermalization time $\tau_T = 1/\Gamma$, with Γ asymptotic value of $\Gamma(t)$, the heating function takes the form

$$\langle n(t)\rangle \simeq \int_0^t \Delta(t_1)dt_1 = \frac{2\alpha^2 KT}{\omega_c}\frac{r^2}{(r^2+1)^2}\Big\{\omega_c t(r^2+1) - (r^2-1)\big[1-e^{-\omega_c t}\cos(\omega_0 t)\big] - re^{-\omega_c t}\sin(\omega_0 t)\Big\}\,, \tag{9}$$

with $r = \omega_c/\omega_0$, α system-reservoir coupling constant, and K is the Boltzmann constant. In deriving the previous equation we have assumed that the initial state of the ion is its vibrational ground state, as it is actually the case at the end of the resolved sideband cooling process [21].

This approximation shows a clear connection between the sign of the diffusion coefficient $\Delta(t)$ and the time evolution of the heating function before thermalization. The diffusion coefficient is indeed the time derivative of the heating function. Since in the high T limit $\Delta(t) \gg \gamma(t)$, whenever $\Delta(t) > 0$ the Master Equation (1) is of Lindblad-type, whilst the case $\Delta(t) < 0$ corresponds to a non-Lindblad-type Master Equation. From Eq. (9) one sees immediately that while for $\Delta(t) > 0$ the heating function grows monotonically, when $\Delta(t)$ assumes negative values it can decrease and present oscillations [22].

To better understand such a behavior we consider three exemplary values of the ratio r between the reservoir cut-off frequency and the system oscillator frequency: $r \gg 1$, $r = 1$ and $r \ll 1$. The first case corresponds to the assumption commonly done when dealing with natural reservoir while the last case corresponds to an engineered "out of resonance" reservoir. For $r \gg 1$ the diffusion coefficient $\Delta(t)$ is positive for all t and r. Therefore the Master Equation is always of Lindblad-type and the heating function grows monotonically from its initial null value. Equation (9) shows that, for times $t \ll \tau_R$, and for $r \gg 1$, $\langle n(t) \rangle \simeq (\alpha^2 \omega_c k T) t^2$, i.e. the initial non-Markovian behavior of the heating function is quadratic in time. For $r = 1$, a similar behavior is observed since also in this case $\Delta(t)$ is positive at all times.

Finally, in the case $r \ll 1$, $\Delta(t)$ oscillates acquiring also negative values. It is worth noting, however, that the long time asymptotic value of $\Delta(t)$ is always positive. Whenever the diffusion coefficient is negative, the heating function decreases, so the overall heating process is characterized by oscillations of the heating function. The decrease in the population of the ground state of the system oscillator, after an initial increase due to the interaction with the high T reservoir, is due to the emission and subsequent reabsorption of the same quantum of energy. Such an event is possible since the reservoir correlation time $\tau_R = 1/\omega_c$, for $r \ll 1$, is much longer than the period of oscillation $\tau_s = 1/\omega_0$. We underline that, although the Master Equation in this case is not of Lindblad-type, it conserves the positivity of the reduced density matrix [17].

4. Experiment for Simulating QBM with Trapped Ions

In the trapped ion context, a high T amplitude reservoir is obtained by applying a random electric field $\vec{E}$ whose spectrum is centered on the axial frequency $\omega_z/2\pi =$ 11.3 MHz of oscillation of the ion [14]. The trapped ion motion couples to this field due to the net charge q of the ion: $H_{\rm int} = -q\vec{x} \cdot \vec{E}$, with $\vec{x} = (X, Y, Z)$ displacement of the c.m. of the ion from its equilibrium position. Remembering that $\vec{E} \propto \sum_i \vec{\epsilon}_i (b_i + b_i^\dagger)$, with b_i and $b_i^\dagger$ annihilation and creation operators of the fluctuating field modes, and that $X \propto \left(a + a^\dagger\right)$ one realizes that this coupling is equivalent to the bilinear one assumed to derive Eq. (1).

The random electric field is applied to the endcap electrodes through a network of properly arranged low pass filters limiting the natural environmental noise but allowing deliberately large applied fields to be effective. This type of drive simulates an infinite-bandwidth amplitude reservoir [14]. It is worth stressing that, for the times of duration of the experiment, namely $\Delta t = 20\ \mu$s, the heating due to the natural reservoir is definitively negligible [14].

The reservoir considered in our paper is a high T thermal reservoir with spectral distribution given by

$$I(\omega) = \frac{2KT}{\pi} \frac{\omega_c^2}{\omega_c^2 + \omega^2} \,. \tag{10}$$

The infinite-bandwidth amplitude reservoir realized in the experiments corresponds to the case $\omega_c \to \infty$ in the previous equation. Therefore, for high T, the reservoir discussed in this paper can be realized experimentally by filtering the random field, used in the experiments for simulating an infinite-bandwidth reservoir, with a Lorentzian shaped low pass filter at frequency ω_c. The change of the ratio r thus would be accomplished simply by changing the low pass filter.

It is well known that non-Markovian features usually occur in the dynamics for times $t \ll \tau_R = 1/\omega_c$. In general, since $\omega_c \gg \omega_0$ and typically $\omega_0 \simeq 10^7$ Hz for trapped ions, this means that deviations from the Markovian dynamics appear for times $t \ll 0.1\ \mu$s. This is the reason why the initial quadratic behavior of the heating function is not observed in the experiments, wherein the typical time scales go from 1 to 100 μs.

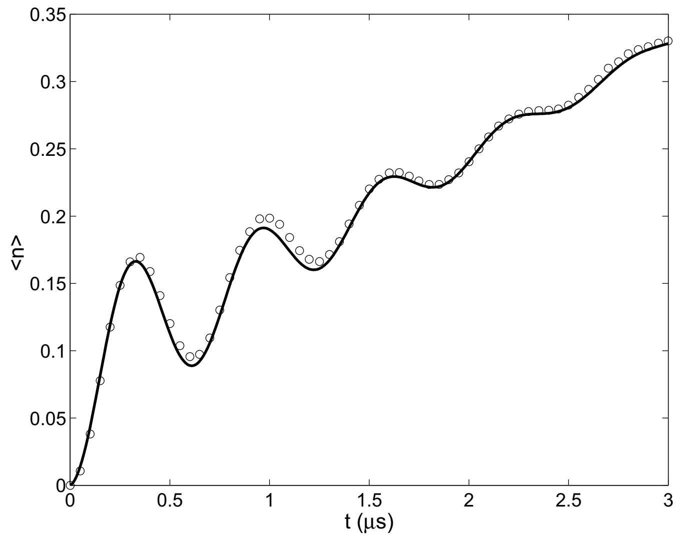

Fig. 1. Time evolution of the heating function for $2\alpha^2 KT/\pi = 0.84 \cdot 10^9$ Hz, $\omega_c = 1$ MHz, $r = 0.1$. Solid line is the analytical and circles the simulation result

A way to force non-Markovian features to appear is to 'detune' the trap frequency from the reservoir spectral density. This corresponds, for example, to the case in which $r = \omega_c/\omega_0 = 0.1$. In this case the reservoir correlation time is bigger than the period of oscillation of the ion and this leads to the oscillatory behavior of the heating function predicted by Eq. (9) for $r \ll 1$. Under this condition $\tau_c = 1\ \mu$s, and therefore the non-Markovian features show up in the time evolution and can be measured. Detuning the trap frequency from the reservoir, however, decreases the effective coupling between the system and the environment and, for this reason, in order to obtain values of the heating function big enough to be measured we need to increase either the coupling constant α^2, which correspond to an increase in the intensity of the voltage applied to the electrodes, or the strength of the fluctuations $\langle V^2 \rangle$, which correspond to an increase in the effective temperature of the reservoir [23]. When these conditions are satisfied, the heating function behaves as shown in Fig. 1.

5. Conclusions

In this paper the dynamics of a single harmonic oscillator coupled to a quantized high temperature reservoir is studied, focusing in particular on the non-Markovian heating dynamics typical of short times. In this regime the system time evolution is influenced by correlations between the system and the reservoir. For certain values of the system and reservoir parameters, virtual exchanges of energy between the system and its environment become dominant. These virtual processes strongly affect the short time dynamics and are responsible for the appearance of oscillations in the heating function (non-Lindblad-type dynamics).

Extending the ideas of using trapped ions for simulating quantum optical systems, a QBM quantum simulator with single trapped ions coupled to artificial reservoirs is proposed. We have carefully analyzed the possibility of revealing, by using present technologies, the non-Markovian dynamics of a single trapped ion interacting with an engineered reservoir, underlining the conditions under which non-Markovian features become observable.

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